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# NOTES AND COMMENTS

## A NOTE ON EXISTENCE AND UNIQUENESS OF EQUILIBRIUM POINTS FOR CONCAVE $N$-PERSON GAMES

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IN [1] J. B. ROSEN GIVES a sufficient condition for the uniqueness of the equilibrium point of a game. The same condition is also sufficient for the stability of the equilibrium point given a reasonable model of player adjustment behavior. The purpose of this note is to show restrictions on the class of games which imply his condition and which may sometimes be of use. In the remainder of the note Rosen's notation and assumptions are used, and it is assumed that the reader is familiar with his paper.

LEMMA: *If each $\phi_i(x)$ is a regular strictly concave function of $x_i$ (i.e., its Hessian is negative definite), each $\phi_i(x)$ is convex in $(x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n)$ and there is some $r > 0$ such that $\sigma(x, r)$ is concave in $x$, then $[G(x, r) + G'(x, r)]$ is negative definite.*

PROOF: Let $H^*(x, r)$, $H^{**}(x, r)$, and $M^k(x, r)$ $(k = 1, \ldots, n)$ be $n \times n$ matrices whose respective elements are $h^*_{ij}(x, r) = \Sigma^n_{k=1} r_k \nabla_{ij}\phi_k(x)$, $h^{**}_{ij}(x, r) = \text{diag}\,(r_1\nabla_{11}\phi_1(x), \ldots, r_n\nabla_{nn}\phi_n(x))$, and $m^k_{ij}(x, r) = r_k\nabla_{ij}\Phi_k(x)$ for $i \neq k, j \neq k$, and $m^k_{ij}(x, r) = 0$ otherwise. Note that $H^*(x, r)$ is negative semidefinite, $H^{**}(x, r)$ is negative definite and each $M^k(x, r)$ is positive semidefinite; hence $[G(x, r) + G'(x, r)] = H^*(x, r) + H^{**}(x, r) - \Sigma^n_{i=1} M^k(x, r)$ is negative definite.

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### REFERENCE

[1] ROSEN, J. B.: "Existence and Uniqueness of Equilibrium Points for Concave $N$-Person Games," *Econometrica*, 33 (1965), 520–534.

[1] The author wishes to acknowledge helpful comments from an unnamed referee.